Optimization Approach To Air Crew Rostering Problem

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INTRODUCTION

Crew cost is an important component of the total cost incurred by an airline in its everyday operations. As a result, the problem of scheduling airline crews and assigning them to appropriate flights is imperative. Prior to explaining the process of crew scheduling, some of the important terminologies used in almost all commercial airlines are explained as follows.

- **Flight Leg:** A flight leg is defined as a single non-stop travel service between two different places such that it is always characterized by a starting airport, destination airport, starting time and destination time.
- **Round Trip:** A round trip is defined as a sequence of flight legs starting at a certain airport and returning back to the same airport. A round trip is also called as a pairing or a rotation.

The term crew in an airline is defined as a group of people assigned to a job in an airline. The job can be serving a flight as attendants, maintenance problems, refueling activities, working in the cockpit of an aircraft during a flight etc. A crew working in the cockpit of an aircraft is termed as a *cockpit crew*, while a crew serving flights as attendants is termed as a cabin crew. This research work takes into account only the cockpit crews in an airline, as our focus is to assign cockpit crews to round trips in the scheduling horizon. A cockpit crew typically consists of a pilot, one or more co-pilots and a navigator. It is assumed that the pilot, co-pilots and navigator in a cockpit crew, stay together throughout the scheduling horizon. The flight attendants, however, do not stay together with a cockpit crew in the scheduling period. Therefore, in several commercial airlines, the assignment of crews to round trips is carried out separately for cockpit crews, cabin crews and other types of crews. *Due to the focus of this paper, the term crew denotes a cockpit crew.*

The problem discussed in this work is called as air crew rostering problem. The problem is to assign cockpit crews to planned round trips so as to generate personalized monthly schedules of crews. This problem is complex, combinatorial and has large dimensions and hence various heuristic algorithms are used to solve it. This paper develops a new mixed integer linear programming model to solve the aircrew rostering problem. The rest of the paper is organized as follows. Section 2 gives a detailed description of our problem and pertinent literature. The MIP formulation of the crew rostering problem is presented in section 3.

LITERATURE REVIEW- AIRCREW ROSTERING PROBLEM

The airline crew rostering problem is a combinatorial optimization problem . This problem has been analyzed by several heuristics in the past three decades. **Buhr (1978)** studied the airline crew rostering and proposed an objective function of minimizing the difference between the average monthly flight time per crew member and the actual monthly flight of a crew member. **Ryan (1992)** modeled the crew rostering problem as a generalized set-partitioning problem and used a linear relaxation and a branch and bound method for solving it. **Gamache et al. (1998)** also modeled this problem as a set-partitioning problem. The solution technique they used for solving this problem was column generation. **Beasley and Cao (1998)** have framed a dynamic programming algorithm for the problem of assigning crews to tasks where the number of crews is not equal to number of tasks. An upper limit on maximum amount of time a crew can work is included in this research. The lower bound of total crew cost is obtained by dynamic programming and this lower bound was improved by **Lagrangean** based penalty procedure and subgradient optimization.

The important terminologies used in this paper with respect to crew and round trip are explained as follows:

Flight time of a round trip is defined as number of flying hours in the round trip. Flight time of a crew is the cumulative sum of flight times of round trips assigned to the crew. Takeoffs of a round trip are the number of takeoff operations

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performed in the round trip. Takeoffs of a crew are equal to the cumulative sum of all takeoff operations performed by the crew in scheduling horizon. Working hours of a round trip is defined as number of hours a crew actually works in the round trip. Working hours of a crew is equal to the summation of working hours of round trips assigned to the crew. In a round trip, working hours constitute the flight time in the round trip and the time required to work on ground. Flight time of a round trip is less than or equal to working hours of the round trip. Spanning time of a round trip is equal to difference between the starting and ending time of the round trip. Spanning time of a crew is the sum of spanning times of round trips assigned to the crew. Spanning time of a round trip encompasses the time a crew actually works in the round trip plus the time the crew does not work in the round trip. The non-working time of a crew may consist of the time required for sleep, relaxation time, etc. Hence, it is obvious that spanning time of a round trip is more than working hours of the round trip.

MATHEMATICAL FORMULATION

The Air Crew Rostering Problem focuses on the assignment of Crews to Round Trips with an objective to minimize the total crew cost. The total crew cost is the cost of crew assignment plus the reserve crew cost. The airline crew rostering problem dealt in this paper is formulated as a zero one mixed integer linear program. In the research work, the researcher has imposed the following constraints on a crew members flying time, takeoffs, working hours, working days and rest period between two consecutive round trips.

It is assumed that all the crew members in a crew (cockpit crew) stay together and have the same roster in a scheduling period.

Let us denote by *m* the total number of crews to be assigned to round trips in a month. The number of round trips in a month is denoted by *n*.

Let us introduce the following binary variables:

$$x_{ij} = \begin{cases} 1 & \textit{if crew i is assigned to roundtrip j.} \\ 0 & \textit{otherwise.} \end{cases}$$

$$x_{i} = \begin{cases} 1 & \textit{if crew i is kept reserve throughout the scheduling horizon.} \\ 0 & \textit{otherwise.} \end{cases}$$

$$x_{i} = \begin{cases} 1 & \textit{if crew i after being assigned to roundtrip j is assigned to roundtrip r} \\ 0 & \textit{otherwise.} \end{cases}$$

$i = 1,\ldots,m.$

$j = 1,\ldots,n.$

$r = 1,\ldots,n.$

Here it can be observed that *j* and *r* both are used to index round trip.

S_j = *starting time of round trip j.*

E_j = *ending time of round trip j.*

Two round trips are considered to overlap each other if their time epochs overlap each other.

$$y_{jr} = \begin{cases} 1 & \textit{if r starts after j and before ending time of round trip j.} \\ 0 & \textit{otherwise.} \end{cases}$$

$w_j = E_j - S_j$ = *spanning time of round trip j.*

D_j = *working hours of round trip j.*

t_j = *number of takeoffs of round trip j.*

d_j = *flight time of round trip j.*

n_j = *number of days involved in round trip j.*

c_{ij} = *cost of assigning crew i of round trip j.*

p_i = *penalty* cost *of crew i.*
T_{jr} = *time required to travel from base of round trip j to base of r.*
ω_{ij} = *weight of crew i with respect toround trip j.*

Here, the term base implies home base or starting and ending airport of the round trip. It is implied that the starting and ending airport of a round trip is same.
The objective function of minimizing the total crew cost is mathematically represented as:

$$Minimize\ z = \sum_i \sum_j c_{ij} . x_{ij} + \sum_i p_i . x_i$$

$$Let\ z_1 = \sum_i \sum_j c_{ij} . x_{ij} = crew\ \cos t\ of\ round\ trips.$$

$$Let\ z_2 = \sum_i p_i . x_i = penalty\ \cos t\ of\ reserve\ crews.$$

$$Let\ z = z_1 + z_2 = Total\ \cos t\ of\ crews.$$

The constraint that each round trip should be assigned one and only one crew is mathematically expressed as:

$$\sum_i x_{ij} = 1, \quad \forall\ j = 1,...,n.$$

The constraints (1), (2) and (3) are the constraints presented by **Lucic and Teodorovic (1999)**. They have been modified in terms of maximum number of flying hours, maximum allowable takeoffs, and maximum allowable number of working hours as per the suggestions obtained by us from Delta Technologies. The total monthly flight time must not exceed 110 hours per crew. This condition is mathematically expressed as follows:

$$\sum_j d_j . x_{ij} \leq 110, \quad \forall\ i = 1,...,m \qquad \mathbf{(1)}$$

The total number of takeoffs per crew per month must not exceed 90. In other words the following relation must be satisfied:

$$\sum_j t_j . x_{ij} \leq 90, \quad \forall\ i = 1,...,m. \qquad \mathbf{(2)}$$

The total monthly number of working hours must not exceed 190 hours per crew. This constraint can be represented as:

$$\sum_j D_j . x_{ij} \leq 190, \quad \forall\ i = 1,...,m. \qquad \mathbf{(3)}$$

The total monthly number of working days must not exceed 22 days per crew. This constraint is given as follows.

$$\sum_j n_j . x_{ij} \leq 22, \quad \forall\ i = 1,...,m. \qquad \mathbf{(4)}$$

The condition for overlap between any two round trips is mathematically represented by the following two constraints.

$$S_r . y_{jr} \leq E_j, \quad \forall\ j = 1,..., n \quad r = 1,..., n \qquad \mathbf{(5)}$$

$$S_r . y_{jr} \geq S_j . y_{jr}, \quad \forall\ j = 1,..., n \quad r = 1,..., n \qquad \mathbf{(6)}$$

If a crew is assigned to two or more round trips, there should not be any overlap between any two round trips assigned to the crew. This constraint can be expressed as follows.

$$x_{ij} + x_{ir} \leq (2 - y_{jr}), \quad \forall\ i = 1,...,m \quad j = 1,...,n \quad r = 1,...,n \qquad \mathbf{(7)}$$

If crew *i* covers round trip *j* and then immediately, round trip *r* as a part of its roster, then the time difference between ending time of round trip *j* and starting time of round trip *r* has to be a minimum of 24 hours plus the time required to travel from home base of round trip *j* to home base of round trip *r*. This constraint can be represented as follows.

$$\{(S_r - E_j) - (24 + T_{jr})\}\left(\sum_i u_{ijr}\right) \geq 0, \quad \forall\ j = 1,\ldots, n \quad r = 1,\ldots, n \qquad \textbf{(8)}$$

A round trip is assigned one and only one crew and the crew after covering the round trip can be immediately assigned to at most one round trip. This constraint is explained as follows.

$$\sum_i \sum_r u_{ijr} \leq 1 \quad , \quad \forall\ j = 1,\ldots, n$$

The relationships between the integer binary variables are mathematically represented by the following constraints. Any two round trips assigned to a crew cannot overlap each other. This condition can be mathematically represented as:

$$\left(\sum_i u_{ijr}\right) + y_{jr} \leq 1 \quad , \quad \forall\ j = 1,\ldots, n \quad r = 1,\ldots, n.$$

If a crew is kept reserve in the scheduling horizon, then any two round trips in the entire scheduling period may or may not overlap each other.

$$x_i + y_{jr} \leq 2 \ , \ \forall\ i = 1,\ldots,m \quad j = 1,\ldots,n \quad r = 1,\ldots, n.$$

A crew may be kept reserve or may be assigned to one or more round trips but not both. This can be mathematically expressed by the following two constraints.

$$x_i + x_{ij} \leq 1 \quad , \quad \forall\ i = 1,\ldots,m \quad j = 1,\ldots, n.$$

$$\left(\sum_r u_{ijr}\right) + x_i \leq 1 \ , \forall\ i = 1,\ldots, m \quad j = 1,\ldots, n.$$

Crew *i* after covering round trip *j* immediately covers at most one round trip.

$$\left(\sum_r u_{ijr}\right) \leq x_{ij} \quad , \quad \forall\ i = 1,\ldots,m \quad j = 1,\ldots,n.$$

The researcher proposes to solve this mixed linear integer programming formulation using the software MPL that uses CPLEX as its solver code. We also report computational results by testing the MIP formulation for small size problems. These results are presented in the following discussion.

Table 1 : Formulation Results

Crews	Round trips	z value
12	45	512
15	45	490
20	45	489
22	45	490
25	45	483

Table 2 : Formulation Results

Crews	Round trips	z value
30	35	367
12	36	433
15	40	442
20	50	550
30	50	528
25	55	584

The above two tables present the numerical results of the mixed integer programming model on the data sets. The objective function value is indicated in the z- value column, which is the total cost of the crew assignment.

CONCLUSIONS AND SUGGESTIONS

The Mixed Integer Programming Problem computes optimal solutions to small size problems. Future work involves developing Neighborhood search heuristics to solve large size problems.

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DISCUSSIONS

☸ Since Reception centre is the primary bottleneck, increasing another server (personnel) would make it to work in a steady state.
☸ The possibility of clubbing functioning of the Reception centre with the registration centre may also be explored since this could cut down one additional node and a total process time of 10 minutes approximately would be required for each patient.
☸ However, it was evidently proved through the simulations that having a single refraction chamber with eight or even six technicians, the hospital could reduce waiting time up to 25 percent, as well as better utilize the resources.

CONCLUSION

Applying queuing models to healthcare processes helps to study variety of parameters like arrival rate, queue length, server utilization and so on. Each parameter could be individually altered to make the system produce high efficiency. Advanced simulations using simulators would help the administrators to see what happens when we change the resources in the system. Modelling can be applied in healthcare, in the areas wherever queue is involved such as rationing, scheduling, bed allocation, laboratory design, and so on.

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